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Common Misconceptions About Absolute Value and Related Thinking Strategies

Awad Faek Altarawneh*[a], & Mohammad Ahmad Alkhateeb[a]

* Corresponding author
Email: a.altarawneh@hu.edu.jo
a. Department of Teaching and Curriculum, Faculty of Educational Sciences, The Hashemite University, Zarqa Jordan

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ABSTRACT

This study aims to identify widespread misconceptions among pre-service mathematics teachers regarding absolute value and their thinking strategies. The study used a mixed-methods approach, combining quantitative descriptive and qualitative analyses. Researchers constructed a comprehension test with thirty-two items based on the concept of absolute value, and 60 mathematics majors from Jordanian universities participated. Interviews were conducted with 15 respondents, purposefully selected, to gain deeper insights and confirm their test responses. Data analysis findings revealed that most participants faced difficulties solving absolute value equations, and their definitions of absolute value varied. Furthermore, six categories of common errors were identified: (i) removing the absolute value symbols; (ii) over-reliance on rules; (iii) converting absolute value symbols into parentheses; (iv) ignoring the numbers inside the absolute value bars; (v) lack of algebraic treatment; and (vi) an inability to graph absolute value. These findings point to important pedagogical errors in teaching certain mathematical topics, suggesting a need for improved instructional strategies.

KEYWORDS

Absolute value; math teachers; common errors; thinking strategies.

INTRODUCTION

To strengthen their teaching abilities, pre-service math teachers must enhance and expand their subject-matter expertise and content knowledge, enabling them to confidently and accurately understand mathematical concepts and solve a variety of mathematical problems. The concept of absolute value, in particular, seems to confuse a substantial number of students. The standard definition of absolute value provided by instructors and textbooks is often stated as "the distance from zero on a number line." However, as Wade (2012) claims, this definition simply offers a geometric interpretation of absolute value, overlooking its broader mathematical implications. When teaching and discussing the concept of absolute value, math instructors often hear students comment that the absolute value of any number is always positive. This oversimplification arises from a lack of comprehensive explanation by the instructor. Students correctly understand that $|4| = 4$ and $|-4| = 4$, and when asked for the absolute value of $|0|$, they respond that it is zero, recognizing that zero is neither negative nor positive. However, they then tend to conclude that absolute value is always zero or positive, a misconception they carry into solving other absolute value problems. For instance, students might incorrectly simplify $|X+3| = 5$ to $X+3 = 5$ or even $|X+3| = -5$, believing negative numbers have no role in absolute value. The lead researcher has observed that students often become confused and frustrated when solving equations like $X+3 = 5$, frequently reformulating it as $X+3 = 5$ or $X+3 = -5$, tackling each case separately, and arriving at two different solutions (2 and −8). This leaves them puzzled, as one result contradicts their belief that absolute value always produces a positive outcome (Ponce, 2008). Math instructors typically teach that absolute value represents the distance between two real numbers on a number line. However, some students tend to oversimplify this concept, believing that absolute value is limited to positive real numbers or a number without any positive or negative sign. It is crucial for math teachers to ensure that students thoroughly understand the concept of absolute value, as it serves as the basis for more advanced topics in calculus, including multiple variables, limits, and continuity. Notably, absolute value is introduced at the higher basic level in most countries' mathematics curricula. Some research suggests that the timing of its introduction creates cognitive challenges in grasping the concept. Although different definitions of absolute value exist, they all stem from more fundamental definitions. In many countries, absolute value is taught through a purely arithmetic approach, resulting in a focus on routine procedures (Abdallah et al., 2018; Almog & Ilany, 2012). In the Jordanian math syllabus for the year 2020, absolute value is introduced in the 11th grade. The key objectives include understanding and analyzing absolute value, applying it to solve real-world problems, and constructing mathematical models that incorporate absolute value in practical contexts. The textbooks aligned with the 2020 syllabus define absolute value as the distance between a number and zero on a number line. Students are guided toward understanding this concept through examples using a step-forward/step-back approach. However, the materials do not emphasise that absolute value is a number without a sign, nor do they explain the process of omitting the absolute value symbol. Various techniques have been developed to help students understand the fundamentals of absolute value without confusion. For example, instructors can use the number line to integrate procedural and conceptual knowledge. (Adinda et al, 2021). Additionally, visual learning is key in mathematics, and teachers can employ graphic representations when teaching absolute value (Pereira et al., 2020). Presenting contextual tasks also provides learners with a meaningful way to fully grasp the concept (Jupri & Gozali, 2021).

Authors in the field of mathematics pedagogy have conducted numerous studies focusing on the difficulties learners face when solving problems related to absolute value (Elia et al., 2016; Pino-Fan et al., 2017; Wilhelmi et al., 2021; Yunita et al., 2019). One possible explanation for these challenges is students' tendency to overgeneralise, combined with the fact that their understanding is often rooted in misconceptions, which they mistakenly regard as the ultimate truth and apply across various contexts.

Research indicates that overgeneralization is a primary factor behind students' misunderstandings in mastering algebra (Aziz et al., 2019; Nisa et al., 2021; Jameson et al., 2023). This issue often stems from instructors' failure to effectively convey the nuances required to fully grasp algebraic rules, particularly when it comes to concepts like absolute value. Rather than building a conceptual understanding, students frequently resort to memorization and procedural strategies, which undermine their ability to truly internalise key ideas like absolute value. This lack of mastery creates a cascading effect across all mathematical areas, especially when students misapply fundamental rules, such as distributive multiplication. The common error of equating absolute value symbols with parentheses shows a deeper issue: educational barriers formed by a misrepresentation of core mathematical principles. These misconceptions become cumulative, and students confront increasing difficulty in overcoming them without first addressing their foundational misunderstandings. Thus, it is crucial for educators to act as facilitators in bridging the gap between students' flawed perceptions and scientifically accurate information, presenting these concepts in a clear, comprehensive manner to rectify the initial misunderstandings.

Teachers' misunderstandings or errors naturally affect students' comprehension of the concept of absolute value. These educational barriers may stem from the "educational contract," a term that refers to the implicit rules formed and negotiated between students and instructors during the learning process. This contract is crucial for helping learners master new concepts and develop problem-solving skills. In the context of teaching absolute value, educational contracting is evident when the absolute value symbol is removed, a procedure that teachers often emphasise. However, if this step is poorly presented, it can cause students to disengage, hindering their ability to fully internalise the concept of absolute value. Students often hold incorrect knowledge, with accompanying strategies that contribute to these mistakes. The way students think undoubtedly affects the presence of errors, regardless of their nature. To correct these misconceptions, it is essential to understand the strategies they employed during their thought processes. This understanding helps educators comprehend how the student arrived at a solution, why they used that approach, and what strategy guided their thinking. By identifying these elements, educators can steer the student towards the correct method, addressing and rectifying their alternative conception. In turn, the student will realise that their flawed reasoning led to the error, prompting them to adopt the correct strategy and eliminate the mistake.

The last two decades have seen several studies published on absolute value (Almog & Ilany, 2012; Çiltaş & Tatar, 2011; Curtis, 2016; Elia et al., 2016; Güveli, 2015; Serhan & Almeqdadi, 2018; Wade, 2012; Wilhemi et al., 2007), all focusing on high school students' depth of understanding of the concept. However, no research has addressed this issue among mathematics students within the Middle Eastern context, despite the fact that misunderstandings acquired at the secondary level often persist into undergraduate studies. The University Education Program for Mathematics Teachers plays a crucial role in addressing these misconceptions, particularly around absolute value, which seems prevalent among students. It is essential that such misunderstandings are resolved before math graduates begin teaching, as their own errors could affect how future students perceive important mathematical concepts. This study is motivated by the need to ensure that secondary school mathematics teachers are fully aware of fundamental concepts like absolute value before they enter the profession. Resolving any potential misconceptions is critical to refining their teaching and ensuring they present the topic accurately to their learners. This study may provide valuable insights for educators in designing effective instructional strategies on the topic of absolute value. It is important to note that calculus courses are not the only ones that emphasise solving absolute value equations, understanding their solutions, and explaining their importance. Several other courses, such as Real Analysis, Number Theory, and Geometry, also require a strong understanding of these fundamental concepts. When students encounter these subjects, they often face significant challenges in solving absolute value equations. Therefore, having a solid foundation in the fundamentals is essential for building the comprehension and problem-solving skills necessary for success in these areas.

This study aims to address the following questions based on the preceding discussion:

- How will do university level mathematics students understand the fundamental concepts of absolute value?
- What common errors do they frequently make when performing tasks involving absolute value?
- How closely are the thinking strategies associated with these common errors aligned with students' perception of absolute value?

METHODS AND PROCEDURES

This study aims to explore how pre-service math teachers conceptualise absolute value. To address the research questions, a quantitative approach was used by calculating the percentage of each type of error made by students in the test. The qualitative aspect involved conducting individual interviews, followed by an analysis to gain deeper insight into the thinking strategies students employed that contributed to their common errors. The study sample was selected using a purposive sampling method, consisting of 60 pre-service mathematics teachers pursuing a bachelor's degree in the mathematics department at a public university in Jordan. All participants were between the ages of 20 and 23, with an average age of 21.8 years. The sample included 40 female students and 20 males, all of whom had successfully completed a calculus course that provided a comprehensive explanation of the concept of absolute value. The researchers conducted in-depth personal interviews with 15 selected learners to gain a deeper understanding of how they formulated their responses to the test questions. The interviewees were selected based on their test scores and categorised into two groups: low and medium achievers. Five participants were selected from the medium group, while 10 were from the low achievers, as they were more likely to make errors when completing the task. Each interview, which lasted between 25 and 35 minutes, was audio-recorded. The interviewer used exploratory questions to prompt both verbal and, when necessary, written responses from the participants, encouraging them to articulate their thinking and strategies during the test. A four-step methodology was employed during the interviews. Each student was asked the following questions:

- How did you arrive at their solutions?
- How would you explain this concept to a colleague?
- What was the rationale behind your choice of approach?
- Who taught you this approach?

In addition, each student was asked to solve a question similar to one from the test and explain their thought process while doing so. The data collected during these interviews were compiled, analysed, and categorised to identify the participants' errors and the strategies they employed.

Two tools were employed in this study: written assessments and in-depth interviews. With input from two specialists in mathematics education, an Absolute Value Comprehension Test was developed, consisting of 32 items. The primary aim was to identify the mistakes made by math students when addressing various questions on absolute value. The test was presented to a specialised mathematics committee to verify its content validity. The committee recommended several modifications, which were subsequently implemented. The test was then administered to a pilot sample of 20 students, not included in the main study, to identify any ambiguous or difficult items and to refine them accordingly. This step also helped determine the appropriate test duration, which was set at 100 minutes. The reliability of the test was calculated utilising the test-retest method, yielding a Pearson coefficient of 0.89. The test was administered in the middle of the 2021/2022 academic year. For the interview process, the content validity of the interview card was confirmed by presenting it to a group of experienced academics, and all suggested modifications were implemented accordingly. The reliability of the interview card was tested by applying it to five students from the pilot sample, all of whom made errors on multiple statements. This step aimed to evaluate the card's effectiveness in revealing students' understanding of absolute value, determine the average time required for the interviews, and assess the students' ability to understand the questions. One week later, the same students were re-interviewed using the same questions and method. Both interviews were transcribed, and the results were compared to assess the consistency of the students' answers. The two analyses were found to be 92% identical. The test itself contained 12 items that required participants to respond to the given data, aimed at evaluating their understanding of absolute value concepts. Participants indicated their responses with one of four options: "I don't know," 'I do not agree', 'I doubt it', and 'I agree.' The remaining 10 items consisted of math problems related to absolute value, most of which were created by the researchers, while the others were adapted from the study by Aziz et al. (2019). Several authors have suggested that learners who struggle with the concept of absolute value often find it difficult to provide an adequate definition. To address this, one of the test statements required students to offer a written definition of absolute value. Four additional statements assessed participants' abilities to solve standard absolute value problems. Students were asked to determine the value of x in equations such as $|X+4|=4$, $|X-5|<0$, $|X|>5$, and $|3X-7|>X$. Three more statements required respondents to solve problems based on discursive reasoning rather than arithmetic operations. For example, students were asked to calculate the value of x for equations like $|X-7|=-3$, $|X+3|+|X-3|=0$, and $|X-4|-|10|=-5$. Another statement asked them to draw a Cartesian graph to demonstrate their understanding of absolute value. In the final statement, participants were required to present a real-world application of the concept of absolute value. Each student completed the test individually, with a total test duration of 100 minutes. The participants' written answers were evaluated to assess their perceptions of absolute value and to identify any errors they made. The interviews were transcribed from the audio recordings, and the data obtained from both the test and interviews were analysed to produce a descriptive analysis. A coding system was applied to the answer sheets to categorise aspects of the concept, the errors committed, and the extent to which participants adhered to the thinking strategies associated with those errors. By emphasizing both qualitative and quantitative data, the study ensured that the results complemented each other, enhancing the reliability, validity, and credibility of the study findings.

FINDINGS

The results of this research were based on data collected from 60 students majoring in mathematics at a Jordanian university. All participants completed a 32-item test developed by experts, focusing on various aspects of absolute value as a mathematical concept. Additionally, 15 students were selected for in-depth interviews to provide more detailed insights into their understanding of the topic. The following sections delve into the specific knowledge and misconceptions these students exhibited concerning absolute value.

The concept of absolute value

Some of the key concepts related to absolute value include zero, the number line, and intervals. A 12-item scale, including an open-ended question, was developed to examine the students' comprehension of these concepts. Table 1 presents the participants' responses, revealing an uncommon consensus regarding the statement that zero is a positive number. Only 28 out of 60 participants answered this correctly by selecting ‘I do not agree’. This basic knowledge is crucial for students to properly understand the concept of absolute value. Items 2, 3, and 4 asked participants to define the absolute value of a real number, and the responses to these items were notably inconsistent. As shown in Table 1, a substantial number of participants incorrectly believed the statement in item 2, ‘The absolute value of a real number is always positive’, to be true, with 58 respondents selecting ‘I agree’. Items 3 and 4 posed similar questions, but participants provided varying responses.

Table 1.

Participants’ responses

No.	Items	I don’t know		I agree		I doubt it		I don’t agree		No responses	
		Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%
1	Zero is a positive number.	0	0.0	27	45.0	3	5.0	28	46.7	2	3.3
2	The absolute value of a real number is always positive	0	0.0	58	96.7	0	0.0	0	0.0	2	3.3
3	The absolute value of a real number is a non-positive number	2	3.3	51	85.0	3	5.0	2	3.3	2	3.3
4	The absolute value of a real number is a positive real number and zero.	0	0.0	27	45.0	8	13.3	22	36.7	3	5.0
5	$\|a\| = \|-a\|$	0	0.0	47	78.3	2	3.3	9	15.0	2	3.3
6	$\|a-b\|$ can be interpreted as the distance between point a and point b on the number line.	14	23.3	25	41.7	8	13.3	8	13.3	5	8.3
7	You can write $\|x-6\| = 13$ as $x-6=13$ or $x-6=-13$.	0	0.0	43	71.7	2	3.3	13	21.7	2	3.3
8	$\|x-2\|=3$ can be represented in the real number line.	2	3.3	26	43.3	16	26.7	14	23.3	2	3.3
9	$\|x-2\|=4$ refers to the case where the distance between x and the point -2 on the real number line is 4.	2	3.3	33	55.0	10	16.7	10	16.7	5	8.3
10	$\|x\|$ refers to the case where the distance between X and zero on the real number line is zero	2	3.3	38	63.3	9	15.0	6	10.0	5	8.3
11	$\|x\| \geq 2$ can be represented on the real number line.	2	3.3	46	76.7	2	3.3	7	11.7	3	5.0
12	$\|x\| < 3$ can be represented on the real number line.	0	0.0	27	45.0	3	5.0	28	46.7	2	3.3

Many reported learning at the high school level that absolute value is always positive, and their teachers had not emphasised that zero is not a positive number. Items 5 and 7 assessed the participants' understanding of certain properties of absolute value, and most students provided correct responses, indicating little confusion regarding these concepts. However, the remaining items (6, 8, 9, 10, 11, and 12) tested the students' ability to associate absolute value with the number line. Responses to these items were inconsistent, with most participants demonstrating a flawed understanding of how to represent absolute value on the real number line. The in-depth interviews revealed that many students had never received a clear explanation of the relationship between absolute value and the number line from their instructors.

Table 2 summarises the participants' responses when asked to define the absolute value of a real number, revealing clear inconsistencies. As expected, the largest proportion of participants (40%) incorrectly claimed that the absolute value of a real number is always positive. Additionally, the responses showed that many participants struggled to differentiate between positive and non-negative numbers (13.3%). Notably, only a small number of students (5 out of 60, or 8.3%) were able to correctly provide the official definition of the absolute value of a real number.

Table 2.

Participants' perceived concepts about the absolute value of a real number

No.	Perceived concepts	Fre.	%
1	The distance on the real number line	8	13.3
2	The non-negative number	2	3.3
3	Use the absolute symbol	2	3.3
4	Positive number	24	40.0
5	Positive number and zero	5	8.3
6	Positive number or non-negative number	3	5.0
7	A number without distinguishing between positive or negative	8	13.3
8	Official definition	5	8.3
9	No responses	3	5.0

Participants' errors and their associated thinking strategies

This section examines the errors participants made in the remainder of the test items. Seven key errors were identified in the written responses provided by the participants. The following subsections offer detailed explanations of each of these errors.

1. Removing the two absolute value bars

Many students made the mistake of believing they could solve problems by simply removing the absolute value bars. While some participants demonstrated the ability to handle common questions, a significant number struggled with this concept. The misconception that absolute value is always positive may have contributed to their tendency to omit or disregard the absolute value symbol, treating the problem as if it were a standard linear equation. In interviews, participants who made this error stated that |a| was equal to a, reflecting a fundamental misunderstanding. This error became more pronounced when students encountered more challenging questions, such as those in Items 4, 5, and 6.

2. Intense focus on rules or strategies

Although several participants stated that the absolute value of a real number is always positive, their responses to items 4, 6, and 8 did not fully support this belief. For question 8, many participants incorrectly thought they could solve the problem by isolating the absolute value bars on both sides. A common strategy they employed was squaring both sides to solve an absolute value equation. For inequalities involving absolute value, most participants relied on familiar strategies, such as: if |x| > a, then x < -a or x > a; and if |x| < a, then –a < x < a. Participants appeared to place excessive trust in these strategies without fully considering the right-hand side of the equation. This over-reliance led them to reach incorrect conclusions in problems like |x-3| < 0 and |x-4|> 2. During interviews, many participants admitted that they focused more on applying the rule to solve the problem rather than engaging with the logic of the right-hand side, even though this approach failed to yield the correct answer.

3. Converting the absolute value bars into parentheses

In their attempts to solve questions involving absolute value, most participants replaced the absolute value sign with dashes or parentheses. However, dashes and parentheses do not serve the same function as the absolute value sign, which follows specific rules. This confusion likely stems from an inadequate understanding of the correct procedures for solving absolute value equations. While it is possible to convert the absolute value sign into dashes after applying the formal definition, a tendency to focus only on the positive or zero case, while overlooking the negative case, may explain why participants treated the absolute value sign as parentheses. One student mentioned that, during high school math classes, he consistently replaced the absolute value bars with dashes but never received feedback from his teachers on this approach. As a result, his misunderstandings persisted unchecked.

4. Moving the number outside the absolute value bars

Another common error observed in the participants' written responses was their tendency to move numbers from inside the absolute value bars to the outside. This strategy was applied consistently across the questions. For example, in the equation |x+5|, some participants incorrectly moved the number 5 outside the bars, rewriting it as |1x|+5. They seemed to believe that the absolute value sign only affects variables like x, rather than the entire expression inside the bars. Similarly, when solving the equation |x-6|, participants incorrectly treated it as |x| + 6. This tendency to move numbers outside the absolute value bars and convert them into positive numbers likely stems from the misconception that absolute values are always positive. Additionally, a misunderstanding of the definition of 'numbers' within the context of absolute value equations may explain why students consistently make such errors.

5. Poor performance in algebra

Participants who successfully mastered the problems related to absolute value were often those with strong skills in working with algebraic expressions. While some students showed a solid understanding of absolute value, their lack of proficiency in algebra led to incorrect solutions, highlighting the need for students to develop both competencies. Several participants in this study struggled with algebraic manipulation. For example, several students attempted to solve the equation $|x+4| = 4$ by squaring only one side of the equation, focusing more on the expression inside the absolute value bars and assuming that removing the absolute value sign was the primary step. As a result, they neglected to square the other side of the equation, leading to incorrect answers.

6. Inefficiency in graphing the function of the absolute value

One of the test items required participants to graph the equation $y=|x-3|$. While most students correctly represented this problem graphically, some made common mistakes. For instance, one participant considered only the positive values of x, ignoring the negative values, resulting in a graph confined to the upper right quadrant of the Cartesian coordinate system. This error likely stemmed from the student's belief that the absolute value of a number is always positive, leading them to plot only the positive values. Additionally, some students attempted to remove the absolute value sign by taking the square of the expression on the right side. This flawed approach resulted in quadratic equations, causing them to graph a quadratic function instead. It appears that these students believed that eliminating the absolute value sign through squaring was the correct initial step in solving such problems. Other students tried to apply the formal definition of absolute value but misused it, defining $y = |x-3|$ as $x = y + 3$ and $x = 3 - y$. This incorrect interpretation led to the construction of different, incorrect lines on the Cartesian plane.

7. Treating inequalities as equations

Some of the written responses from participants showed a tendency to convert inequalities into equations. These students identified the values of the variable and treated them as the solution to the inequality, rather than solving the inequality itself. This may be due to two primary reasons. First, the students likely failed to master the concepts related to inequalities during high school, where these topics are taught mainly in special units during the eighth and 11th grades. Second, these concepts were not reviewed when the student's studied calculus, as they were assumed to be basic knowledge already covered. This is particularly relevant since these and other foundational topics are often included in Calculus 101 during the first semester at university. Another possible explanation is that professors may perceive these concepts as straightforward for students, particularly those from the scientific stream in secondary education who are accepted into the mathematics major. This finding aligns with the study by Aziz et al. (2019). Table 3 presents the percentages of thinking strategies associated with erroneous concepts of regular and decimal fractions and operations on them.

Table 3.

Percentages of thinking strategies associated with erroneous absolute value concepts

No.	Strategies	Percentage
1	Removing the two absolute value bars	94%
2	Intense focus on rules or strategies	92%
3	Converting the absolute value bars into parentheses	87%
4	Moving the number outside the absolute value bars	82%
5	Poor performance in algebra	76%
6	Inefficiency in graphing the function of the absolute value	61%
7	Treating inequalities as equations	48%

Adherence to the thinking strategies associated with common erroneous concepts of absolute value

This issue was explored further through questions posed during the in-depth interviews. To evaluate the participants' consistency and determination in sticking to the same problem-solving approaches, several questions analogous to the test problems were asked. Each student's interview responses were then analysed, classified, and compared with their test answers to determine the degree of consistency over time. The findings revealed that a substantial number of students stuck to the same solution strategies that had led to common errors on the test, showing a persistence in their misconceptions.

91% of the participants continued to incorrectly remove the absolute value bars during the interview. For instance, when one student, who had previously solved the exam question by mistakenly removing the absolute value signs, was asked to solve the equation $|3 - x| = 5$, they again incorrectly removed the absolute value signs and rewrote the equation as $3 - x = 5$, failing to account for the two possible cases for the absolute value.90% of the participants continued to rely heavily on memorised rules or strategies. In one interview, when a student who had used this approach was asked to solve the equation $|2 + x| = |6 - 3x|$, they applied only one rule—removing the absolute value signs directly—without considering the different possible cases. The student proceeded with the following solution: $2 + x = 6 - 3x$, and then solved it as: $3x + x = 6 - 2$, $4x = 4$, $x = 1$. This solution shows the student's failure to recognise the need to consider multiple cases when solving absolute value equations.71% of the participants continued to use the strategy of converting the absolute value signs into dashes or parentheses. During the interview, one student who had previously made this error on the exam was asked to solve the equation $|5 - x| + 3 = 10$. The student incorrectly replaced the absolute value signs with parentheses and wrote the equation as $(5 - x) + 3 = 10$. They then proceeded with the solution as follows: $(5 - x) = 7$, $- x = 7 - 5$, $- x = 2$, $x = -2$. This error highlights a fundamental misunderstanding of how to approach absolute value equations, as the student failed to account for the two possible cases that should be considered when dealing with absolute value.90% of the participants continued to adhere to the strategy of incorrectly moving the number outside the absolute value sign. During the interview, one student, who had previously made this error on the exam, was asked to solve the equation |3x - 9| = 12. The student incorrectly transformed the equation to 3|x - 3| = 12, and then proceeded to solve it as |x - 3| = 4. This error shows a misunderstanding of how to properly work with absolute value equations, as the student incorrectly factored out the coefficient (3) from inside the absolute value, , neglecting the correct approach for solving such problems.

Furthermore, an analysis of the students' interview responses showed that 84% of the learners demonstrated poor mastery of algebra skills. In one interview, a student with weak algebra skills was asked to solve the equation |2x - 4| = 8. The student incorrectly solved it by dividing the entire absolute value expression by 2: (|2x - 4|) ÷ 2 = 8 ÷ 2, and then continued from there. This error reflects a fundamental misunderstanding of how to manipulate absolute value equations and algebraic expressions. Dividing the absolute value expression instead of considering both possible cases for the absolute value reveals a lack of understanding of algebraic concepts and the correct structure of absolute value equations.

Moreover, The findings showed 79% of the participants lacked proficiency in graphing the absolute value function, and 75% consistently treated inequalities as if they were equations. In one interview, a student who made this error was asked to solve the inequality |2 - x| < 5. The student mistakenly approached it as an equation: 2 - x = 5, and proceeded to solve from there. This shows an important misunderstanding of how to properly handle inequalities involving absolute values. By treating the inequality as an equation and failing to account for the two cases— (2 - x < 5) and (2 - x > -5)—the student was unable to reach the correct solution. This error highlights the need for more targeted instruction on the differences between solving equations and inequalities, especially in the context of absolute values. Table 4 shows the percentages of reliability for thinking strategies associated with erroneous concepts of absolute value.

Table 4.

Percentages of reliability of thinking strategies associated with erroneous concepts of absolute value

No.	Strategies	Reliability percentage
1	Removing the two absolute value bars	91%
2	Intense focus on rules or strategies	90%
3	Converting the absolute value bars into parentheses	71%
4	Moving the number outside the absolute value bars	90%
5	Poor performance in algebra	84%
6	Inefficiency in graphing the function of the absolute value	79%
7	Treating inequalities as equations	75%

DISCUSSION

The primary objective of this study was to survey Mathematics students, who are potential secondary school teachers, to evaluate their understanding of the absolute value concept through their written responses to a specially designed test. This test served as the primary data collection tool and included questions on the meaning of absolute value, along with other tasks related to it. The findings revealed that participants struggled considerably with the tasks. When approaching tasks related to absolute value, they made numerous errors, many of which were consistent with those identified in previous research. Several sources of these errors were identified. Absolute value, defined as a non-negative number—either zero or positive—was often misunderstood by participants, who equated non-negative with positive. This misinterpretation stems from a poor understanding of zero, which, according to the participants, was not considered a positive number. Previous studies (Jupri & Gozali, 2021; Ponce, 2008; Taylor & Mittag, 2015) have similarly shown a lack of understanding in this area. More concerning is that many teachers in schools either do not correct this misconception or are unaware that it needs correcting. It is essential to address this issue at the beginning of math lessons, as doing so would help students gain a stronger understanding of absolute value—a crucial concept in many other areas of mathematics. As a result, we now have a deeper understanding of how participants perceive the absolute value of a real number. The students provided conflicting responses on this issue. While the majority insisted that the absolute value of any number is positive, only a few were able to present the formal definition. This may be because, in Jordan, it is common for academics to interpret the absolute value of a real number as positive, a view shared by both teachers and students. This misunderstanding is further compounded by the fact that most students do not consider zero as a positive number. Literature suggests a discrepancy in how the concept of absolute value is perceived across different countries. For instance, Turkish students generally understand absolute value as the distance from zero. Similarly, research conducted in Cyprus by Gagatsis and Panaoura (2014) reported that many students there view absolute value as numbers without mathematical signs. It is clear that the educational process in each country considerably affects students' understanding of absolute value. Additionally, the flawed comprehension of math teachers regarding this topic contributes to the persistence of inaccuracies, as these errors are passed down from one generation of math instructors to the next. Graphs and real number lines can effectively communicate the concept of absolute value both graphically and externally, helping students internalise the idea in a meaningful way. By visualizing it, students may find the concept of absolute value easier to grasp. However, the participants' responses show that there is no clear relationship between their understanding of absolute value concepts and their internal representations. This disconnect may be due to students' flawed comprehension of the meaning and flexibility of external representations. The findings of this study reveal that while most participants were able to create graphic representations of the absolute value function $y=|x-3|$, confusion was widespread when they were asked to handle the symbolic representation or explain the meaning of absolute value. This suggests that being able to graph the absolute value function does not necessarily equate to a clear and accurate understanding of the concept. Therefore, mathematics teachers must help students make sense of various external representations of absolute value by incorporating these representations and their formal definitions into their teaching. Numerous studies (Aziz & Kurniasih, 2019; Aziz et al., 2019; Nisa et al., 2021; Mosia et al., 2023) have emphasised the importance of utilising external representations as a means of guiding students towards a clearer and more comprehensive understanding of this frequently misunderstood mathematical concept. Study participants frequently made errors such as omitting the absolute value sign, focusing solely on strategy rules, turning the absolute value sign into ‘brackets’, mishandling algebraic equations, and struggling to design absolute value functions. These errors mirror those identified in numerous previous investigations. Existing studies have shown students' misunderstandings and common mistakes when tackling absolute value problems. Some have reported that students' errors include logical inconsistencies, omission of the absolute value symbol, indistinguishability, and other similar mistakes (Almog & Ilany, 2012; Sierpinska et al., 2011). Research by Gagatsis and Panaoura (2014) and Kumari (2021) similarly reported that a frequent, stereotypical error among students is their tendency to eliminate the absolute value bars—a mistake observed in students from many different countries. This fundamental misunderstanding of the basics leads them to adopt this faulty strategy without considering the concept as part of the educational framework.

In Jordanian mathematics classrooms, the most prevalent educational contract frames the absolute value as a number always considered positive. This perception has become deeply ingrained in students' mathematical understanding and may lead to some misunderstandings. Such misconceptions contribute to common errors in solving absolute value problems, including ignoring the absolute value bars, focusing solely on rules or strategies, incorrectly moving numbers from within the absolute value sign, and making mistakes when graphing absolute value functions.

The conversion of the absolute value sign into parentheses stems from students' tendency to overgeneralise. Kumari (2021) and Aziz et al. (2018) claimed that this overgeneralization arises from the direct interpretation of a specific idea without considering related mathematical concepts. As a result, students interpret absolute value bars as parentheses, without reflecting on the true definition of absolute value. Additionally, correctly solving such problems requires a certain level of proficiency with algebraic expressions. The findings of this study suggest that understanding the concept of absolute value does not necessarily equate to solving problems involving it successfully, and the opposite is also true. Therefore, before introducing the topic of absolute value, mathematics instructors must ensure that students have a solid foundation in algebra. Mastery of both the absolute value concept and the skills required to solve algebraic expressions is key to success. The interview results revealed various thinking strategies associated with students' errors in understanding the concept of absolute value. The most prominent were the removal of absolute value signs and an overemphasis on memorizing laws or procedures. This can be attributed to students' tendency to adopt a mechanical approach to solving mathematical problems, where they prioritise memorising rules and procedures over truly understanding them. Consequently, they overlook the conceptual aspect needed in certain problems, such as recognising that absolute value represents the distance of a number from zero, regardless of its sign (Cahyani et al., 2024). Additionally, an excessive focus on quick solutions and the application of general rules in education may encourage students to adopt superficial strategies. When students are taught to handle equations and mathematical laws in a highly technical manner, they may fail to grasp the conceptual foundations governing these laws in specific contexts, such as absolute value (Baştürk, 2023).

The lack of training in critical thinking and exploring alternative solutions may also contribute to students' reliance on quick, memorised solutions rather than engaging with the deeper meaning of the problem. Providing more opportunities for conceptual thinking and practicing diverse strategies could help reduce these errors and enhance students' understanding of the absolute value concept. This finding aligns with the study by Khoerunnisa et al. (2024), which examined how students approach the concept of absolute value. The strategy of converting the absolute value sign into parentheses, extracting numbers from within the absolute value, and poor algebra performance reflects a superficial understanding of the concept. Many students lack a deep comprehension of absolute value, leading them to adopt inappropriate strategies, such as treating the absolute value sign as parentheses or as a simple algebraic element without recognising its conceptual importance. Often, students are taught to solve problems through mechanical steps and procedures, with insufficient emphasis on fundamental concepts. As a result, they rely on memorised strategies like "extracting the number from within the absolute value" without truly understanding the underlying problem (Nisa et al., 2021). The study by Fitzsimmons et al. (2020) attributed this issue to a lack of conceptual guidance on the importance and meaning of absolute value, particularly in the context of handling negative and positive numbers. When students are not sufficiently trained to consider the deeper mathematical meanings behind symbols, they are more prone to errors. Additionally, challenges with algebra play an important role, as Jupri and Gozali (2021) reported that many students struggle with algebra in general, which negatively impacts their ability to solve problems involving absolute values. This was further supported by Wilhelmi et al. (2021). Moreover, this study uniquely identified strategies not previously addressed in earlier research, such as students' lack of proficiency in graphing absolute value functions and their tendency to treat inequalities as if they were equations. The results showed that more than 75% of the students interviewed adhered to problem-solving strategies that led to errors in their understanding of absolute value between the first and second interview attempts. It was observed that the students repeated the same mistakes made during the test and reproduced these errors in the interviews, indicating that their responses were systematic and based on fixed principles and beliefs. This suggests that these strategies were not random but had a deeper cognitive structure. Piaget emphasised that the confusion and frustration students experience when learning new concepts may arise from a conflict between prior knowledge and new knowledge. If the new knowledge is inconsistent with or disconnected from prior knowledge, learners often resort to memorisation. However, when they later try to recall the concept, they remember it only partially and inaccurately, leading to errors. This finding aligns with the results of studies by Gagatsis and Panaoura (2014) and Jupri and Gozali (2021).

CONCLUSION

This study aimed to underscore the need for further research into the misconceptions commonly held by students aspiring to become math teachers, particularly regarding the concept of absolute value and the errors they consistently make when solving mathematical problems. Based on the results, it is evident that most participants mistakenly believe that absolute value is always a positive number. Strong evidence suggests that teachers need to reconsider their understanding of zero before addressing absolute value. Several common errors must be addressed to ensure that the concept of absolute value is taught correctly in schools and universities. These errors include students (i) ignoring the absolute value sign, (ii) focusing too intensely on rules or strategies, (iii) converting absolute value bars into parentheses, (iv) performing poorly in algebra, and (v) lacking proficiency in graphing absolute value functions.

The researchers propose several implications for improving second-stage math instructors' understanding of absolute value before they begin teaching. It is essential that they gain a deep mastery of this important mathematical concept to help new learners understand it. Educators must ensure that, prior to entering any educational institution, teachers (1) understand the concept of zero, (2) can provide an accurate definition of absolute value, (3) are proficient in performing algebraic operations, and (4) can convert between different representations of the absolute value function. These points should be prioritised when developing an activity-based teaching programme.

The researchers acknowledge several limitations of this study. Firstly, it was conducted on a small scale, involving a relatively small sample of 60 pre-service secondary school mathematics teachers from a single university. Secondly, the primary data collection tool was a relatively brief 32-item written test, supplemented by 15 in-depth interviews conducted with selected participants from the written test.

The researchers suggest that further studies are needed on this subject, as it has been shown that most pre-service teachers lack a clear understanding of absolute value. A major unresolved issue is how educators approach teaching absolute value within the educational process. It is evident that this sample of aspiring math instructors failed to grasp these topics adequately, likely due to the teaching methods used by in-service educators. Therefore, closely observing how absolute value is taught could provide valuable insights for educators.

Additionally, observing secondary school mathematics teachers could further enhance understanding by examining the parallels and discrepancies in how absolute value is taught at the secondary and higher education levels.

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