



Cryptarithmic as an Epistemic Task: Revealing Elementary Students' Numerical Reasoning Under Constraint-Based Problem Solving

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ABSTRACT

Elementary mathematics learning rarely provides opportunities to observe how students reason numerically when solutions cannot be obtained through routine procedures. This study conceptualizes cryptarithmic as an epistemic task that reveals elementary students' numerical reasoning under constraint-based problem-solving conditions. Rather than evaluating correctness, the study focuses on how students coordinate numerical constraints, justify decisions, and regulate reasoning in the presence of interacting constraints. Using a mixed-method sequential explanatory design, the study involved 27 fifth-grade students and 4 mathematics teachers from an Indonesian elementary school. Students engaged with a cryptarithmic task set designed to externalize numerical reasoning, followed by interviews with students and teachers. Quantitative performance patterns were interpreted as epistemic signals, while qualitative data illuminated reasoning processes, epistemic breakdowns, and mediation across perspectives. The findings reveal substantial variation in students' numerical reasoning orientations, indicating identifiable epistemic thresholds as constraint interaction increases. A key result shows that three-digit addition produced greater epistemic difficulty than two-digit multiplication, demonstrating that constraint density, rather than operation type, governs reasoning complexity. Teachers interpreted cryptarithmic as a diagnostic task for reasoning, whereas students experienced it as an epistemically demanding challenge, revealing both convergence and divergence across perspectives. These results demonstrate that cryptarithmic functions as a powerful epistemic task that makes numerical reasoning visible through patterns of coherence, breakdown, and regulation, offering a theoretically grounded lens for studying constraint-based reasoning in elementary mathematics.

KEYWORDS

Cryptarithmic; epistemic task; numerical reasoning; constraint-based problem solving; elementary mathematics.

INTRODUCTION

In mathematics education, increasing attention has been directed toward higher-order thinking skills as essential outcomes of schooling (Di et al., 2019; Yumiati et al., 2025). Yet, an enduring concern remains regarding what such skills actually entail in elementary mathematics classrooms. While students may demonstrate procedural accuracy in arithmetic, their engagement with mathematical knowledge often remains limited to executing algorithms rather than constructing, justifying, and evaluating numerical claims (Schoenfeld, 2013). This tension highlights a fundamental epistemic problem in early mathematics education: students may learn how to calculate without learning how to know mathematically.

International assessments and classroom-based studies consistently indicate that many elementary students struggle when confronted with non-routine problems that require interpretation, constraint coordination, and justification. Rather than engaging in systematic reasoning, students frequently rely on guessing, fragmented trial-and-error strategies, or surface-level cues, particularly when problems cannot be solved through familiar procedures (Mullis et al., 2020; Phonapichat et al., 2014; Sudirman et al., 2025; 2026). Such patterns of difficulty are further documented in studies on learning obstacles in elementary and secondary mathematics, where students' limited conceptual grounding prevents them from interpreting and constructing mathematical meaning beyond procedural steps (Isnawan et al., 2024). These patterns suggest that the central challenge is not merely insufficient skill, but limited opportunities for students to engage in epistemic activity, activity in which mathematical solutions must be warranted, validated, and regulated through reasoning.

From an epistemic perspective, learning mathematics involves developing ways of knowing: understanding what counts as a valid solution, how certainty is established, and how conflicting possibilities are resolved (Schoenfeld, 2013; Yumiati et al., 2024). Tasks that support such engagement must do more than require correct computation; they must obligate learners to confront constraints, test assumptions, and justify decisions. However, many conventional arithmetic tasks permit success through rote procedures, thereby obscuring students' underlying numerical reasoning and masking differences between superficial correctness and epistemically grounded understanding (Cai & Nie, 2007; Foster, 2023; Zhou et al., 2023).

Three interrelated concepts anchor the theoretical orientation of this study. An epistemic task is defined as a problem-solving activity that obliges learners to construct, justify, and evaluate knowledge claims rather than reproduce procedures — it is characterized not by surface difficulty, but by the extent to which it compels reasoning about the validity of mathematical decisions (Muis et al., 2015; Foster, 2023). Epistemic engagement refers to the quality of cognitive involvement through which learners actively interrogate constraints, test assumptions, and seek justification for their conclusions, extending beyond behavioral participation to encompass reflective evaluation of solution coherence (Chinn et al., 2011; Greene et al., 2010). Epistemic regulation, in turn, denotes the metacognitive processes through which learners monitor consistency, detect contradiction, and revise reasoning when prior

decisions produce logical conflicts — the governing mechanism that either sustains or disrupts engagement under uncertainty (Muis et al., 2015; Winne & Hadwin, 2008).

These three constructs operate as an integrated system. An epistemic task creates the structural conditions that make engagement necessary by embedding constraints that prevent procedural shortcuts. Epistemic engagement is the observable manifestation of how a learner responds to those demands — whether the problem is treated as a knowledge-construction challenge or reduced to unsystematic trial. Epistemic regulation governs whether that engagement is sustained: when functioning, it enables learners to coordinate constraints globally and revise assumptions coherently; when it breaks down, reasoning collapses. This framework is grounded in epistemic cognition research (Hofer & Pintrich, 1997; Muis et al., 2015) and extends cognitive load theory (Sweller, 1988) beyond information quantity to encompass the relational complexity of interacting constraints — a form of epistemic load particularly relevant to constraint-based arithmetic tasks, as elaborated in the following section. Cryptarithmic tasks present a distinctive opportunity to address this limitation. In cryptarithmic, digits are replaced by symbols, and solvers must determine consistent numerical assignments that satisfy multiple interdependent constraints (Namdeo et al., 2012). Crucially, no solution can be reached through direct calculation alone. Progress requires the coordination of place value, operational rules, and logical consistency across the entire structure of the problem. Every numerical decision must be justified in relation to the constraints imposed by the task. In this sense, cryptarithmic constitutes a form of constraint-based problem solving that inherently demands epistemic engagement: solvers must reason about why certain assignments are possible, why others are not, and how validity is established within the problem space (Isuzugawa et al., 2021).

Despite these epistemic affordances, cryptarithmic has been largely positioned in prior research as a pedagogical novelty, enrichment activity, or motivational tool rather than as a vehicle for examining mathematical reasoning. Studies at the elementary level have predominantly focused on performance outcomes or affective responses, often situating cryptarithmic within game-based or competitive instructional frameworks (Widodo et al., 2019). Although such approaches demonstrate that cryptarithmic can be both engaging and instructionally effective, they leave largely unaddressed the more fundamental question of what cryptarithmic reveals about how elementary students reason numerically when solutions cannot be obtained through routine procedures.

More broadly, research on innovative mathematics instruction frequently emphasizes effectiveness, whether a method improves achievement or higher-order thinking skills, rather than examining the epistemic mechanisms through which students construct and validate mathematical knowledge (Ahdhianto et al., 2020; Zhou et al., 2024). Even multi-perspective studies that incorporate teacher and student viewpoints tend to frame these perspectives pedagogically or affectively, without explicitly connecting them to students' forms of reasoning

and justification. As a result, the epistemic dimension of elementary problem solving remains under-theorized.

Responding to this gap, the present study reframes cryptarithmic not as an instructional technique, but as an epistemic task that makes students' numerical reasoning visible under conditions of constraint-based problem solving. By analyzing how elementary students approach cryptarithmic tasks, how they justify and validate numerical assignments, and how different forms of reasoning relate to success and breakdown, this study seeks to illuminate the epistemic nature of students' engagement with arithmetic. A mixed-method, multi-perspective approach is employed to integrate students' problem-solving performance, students' articulated experiences, and teachers' interpretations, allowing for a comprehensive examination of reasoning processes rather than outcomes alone.

Accordingly, this study is guided by the following research questions: (1) What forms of numerical reasoning do elementary students exhibit when solving cryptarithmic tasks under constraint-based conditions? (2) How do students establish and justify the correctness of their solutions when direct computation is insufficient? (3) What is the relationship between students' forms of numerical reasoning and their success or failure in cryptarithmic problem solving? (4) In what ways do student reasoning patterns, student experiences, and teacher perspectives converge or diverge in their interpretation of cryptarithmic as an epistemic task? Through this epistemic framing, the study contributes to mathematics education research by demonstrating how cryptarithmic can function as a powerful analytical lens for understanding elementary students' ways of knowing in mathematics, beyond what is accessible through conventional arithmetic problems.

METHODS

Research Design

This study employed a mixed-method sequential explanatory design grounded in an epistemic research logic to investigate cryptarithmic as an epistemic task for revealing elementary students' numerical reasoning under constraint-based problem solving. This design was theoretically motivated by the assumption that students' ways of knowing in mathematics cannot be adequately inferred from performance outcomes alone. While quantitative data can indicate patterns of success, difficulty, and variation across tasks, epistemic reasoning, such as how students justify solutions, coordinate numerical constraints, and regulate certainty, requires interpretive access to learners' explanations and reflections (Creswell & Clark, 2017; Schoonenboom & Johnson, 2017).

In the first phase, quantitative data were collected through cryptarithmic tasks designed to elicit variation in students' performance under different constraint conditions. These data were used to map the epistemic landscape of students' problem solving, identifying patterns of correctness, breakdown, and sensitivity to digit complexity. Quantitative findings were not treated as final indicators of effectiveness, but as epistemic signals that guided the

focus of the qualitative phase. In the second phase, qualitative data were collected through semi-structured interviews with students and focus group discussions with teachers to examine the reasoning processes underlying the observed performance patterns, including students' justification strategies and teachers' interpretations of student reasoning.

Beyond methodological sequencing, the research design was informed by the view that epistemic reasoning is task-dependent and context-sensitive, emerging through interaction between learners and the constraints embedded in a problem. Accordingly, cryptarithmic tasks were treated as designed epistemic contexts rather than neutral assessment items. Integration of quantitative and qualitative phases enabled the development of epistemic meta-inferences by examining convergence and divergence between performance outcomes, students' reasoning processes, and teacher perspectives. This design ensured that conclusions about cryptarithmic as an epistemic task were grounded in multiple forms of evidence and supported a theoretically informed understanding of elementary students' numerical reasoning under constraint-based conditions.

Participants

This study was conducted in an urban elementary school in Jakarta, Indonesia, selected purposively to support the epistemic aims of the research. The selection criteria included: (1) the school's openness to instructional innovation, (2) availability of adequate learning facilities to support non-routine mathematical tasks, (3) willingness of teachers and students to participate in in-depth educational research, and (4) representativeness of typical urban elementary school characteristics in Indonesia. This context was considered appropriate for examining students' numerical reasoning under constraint-based problem-solving conditions. The research population comprised all fifth-grade students enrolled in the 2024/2025 academic year ($N = 135$) and all fifth-grade mathematics teachers ($N = 6$). From this population, 27 fifth-grade students were selected using purposive sampling based on variation in mathematical proficiency (high, medium, and low) with a 1:2:1 ratio. This sampling strategy was epistemically motivated, aiming to capture qualitative differences in students' ways of reasoning rather than to achieve statistical representativeness (Patton, 2015). Variation in proficiency was essential for identifying contrasting forms of epistemic engagement with cryptarithmic tasks. The demographic characteristics of the student participants are summarized in Table 1.

Table 1.

Demographic Characteristics of Student Participants (n = 27)

Characteristic	Description
Grade Level	Grade 5 (Academic Year 2024/2025)
Age Range	10 – 11 Year
Gender	9 males, 18 females
Mathematical proficiency	High (n = 7, 25.9%), Medium (n = 8, 29.6%), Low (n = 12, 44.5%)

In addition, four fifth-grade mathematics teachers were included using total sampling due to the small population size and the need to obtain comprehensive pedagogical interpretations of students' reasoning processes. Their demographic characteristics are presented in Table 2.

Table 2.

Demographic Characteristics of Teacher Participants

Characteristics	Description
Age Range	40 – 56 years
Gender	4 females
Teaching Experience	5 – 15 years

For the qualitative component, student interviews were conducted until theoretical saturation was reached, indicated by the absence of substantively new insights into students' reasoning and justification patterns. For the quantitative component, the sample size of 27 students was sufficient for descriptive statistical analysis and for categorizing performance patterns that served as epistemic indicators rather than inferential claims (Cohen et al., 2018).

Data Collection

Data were collected using three coordinated instruments designed to examine cryptarithmic as an epistemic task and to capture students' numerical reasoning from multiple perspectives. Each instrument was selected to target a distinct layer of epistemic evidence: task performance to externalize reasoning patterns, teacher interviews to capture pedagogical interpretation, and student interviews to access lived reasoning experiences directly.

The first instrument was a Cryptarithmic Task Set administered to all 27 participating students during regular mathematics class sessions at the research site. The task set functioned as an epistemic probe rather than a test of computational accuracy, designed to externalize how students coordinate digit relationships, place value, and operational constraints under conditions where direct calculation is insufficient. It consisted of four cryptarithmic problems with increasing structural complexity — one easy, two medium, and one difficult — covering two-digit addition, three-digit addition, two-digit subtraction, and two-digit multiplication. These task structures were selected to vary constraint density and interaction, enabling observation of reasoning coherence and breakdown as epistemic demands increased. Students completed the tasks individually and in writing, without access to calculators or reference materials, within a single 60-minute session. Task administration was supervised by the research team to ensure standardized conditions across all participants. Content validity was confirmed through expert review by two mathematics education specialists (CVI = 0.89), and internal consistency reliability analysis yielded a Cronbach's alpha of 0.78, indicating adequate reliability for an exploratory instrument of this type (Field, 2018).

Task outcomes were organized in tables that functioned as epistemic representations, externalizing patterns of numerical reasoning rather than merely summarizing scores. The

second instrument consisted of semi-structured interviews conducted with all four participating mathematics teachers through a focus group discussion format. The focus group was held after students had completed the cryptarithmic tasks, allowing teachers to reflect on their observations of students' reasoning processes during task engagement. Each focus group session lasted approximately 60 minutes and was audio-recorded with participants' consent. Interview questions focused on teachers' familiarity with cryptarithmic, perceived epistemic usefulness, observed student difficulties, and recommendations for classroom implementation. The focus group format was selected to encourage collegial discussion and to surface divergent pedagogical interpretations that might not emerge in individual interviews (Creswell & Clark, 2017).

Figure 1.

Cryptarithmic Test

1.	$\begin{array}{r} AA \\ + BA \\ \hline 98 \end{array}$ <p>What are the correct values of A and B?</p>
2.	$\begin{array}{r} ACB \\ + ACB \\ \hline BDDC \end{array}$ <p>What are the correct values of A, B, C, and D?</p>
3.	$\begin{array}{r} 54 \\ - AB \\ \hline 1B \end{array}$ <p>What are the correct values of A and B?</p>
4.	$\begin{array}{r} AB \\ \times A \\ \hline 96 \end{array}$ <p>What are the correct values of A and B?</p>

The third instrument consisted of individual semi-structured interviews conducted with six purposively selected students, chosen to represent variation across the three performance profiles identified in the quantitative phase. Interviews were conducted after the quantitative analysis was complete, ensuring that student selection was guided by the epistemic patterns observed in task performance. Each interview lasted approximately 30–45 minutes and was audio-recorded with student and parental consent. Interview questions focused on students' reasoning strategies, perceived task difficulty, emotional responses during problem solving, and criteria used for determining solution correctness. Interviews were conducted in Bahasa Indonesia to ensure participants could articulate their reasoning without language barriers, and transcripts were subsequently translated into English for analysis. Together, these three instruments operated as complementary epistemic probes: task performance externalized reasoning, teacher discussions illuminated pedagogical mediation, and student interviews provided direct access to lived experiences of epistemic engagement and regulation.

Data Analysis

Quantitative Analysis: Performance Patterns as Epistemic Signals

Students' performance on cryptarithmic tasks was analyzed using descriptive statistics, including mean, median, mode, standard deviation, and frequency distributions (Field, 2018). In

line with the epistemic focus of the study, these statistics were not treated as indicators of learning effectiveness, but as epistemic signals reflecting how students responded to varying numerical constraints embedded in cryptarithmic tasks. Descriptive analysis was selected due to the exploratory nature of the study, the relatively small sample size, and the emphasis on identifying patterns of variation rather than conducting inferential comparisons (Pallant, 2020). All analyses were conducted using SPSS version 28.0 to ensure computational accuracy and clarity of representation.

To support epistemic interpretation, students were categorized into performance-based groups using a normal distribution criterion ($\mu \pm 0.5\sigma$). This criterion was selected for three reasons. First, it is grounded in normal distribution theory, which assumes that performance scores in a sufficiently varied sample approximate a bell curve, making standard deviation a principled basis for identifying meaningful differences in performance levels (Field, 2018). Second, the $\pm 0.5\sigma$ threshold has been applied in prior mathematics education research as a balanced partition that avoids the extremes of overly broad categories (e.g., $\pm 1.0\sigma$, which collapses most students into a single middle group) and overly narrow ones that produce unreliably small subgroups (Azwar, 2012; Cohen et al., 2018). Third, given the exploratory and epistemic orientation of this study, the purpose of categorization was not to rank students by ability but to identify contrasting reasoning orientations — a goal for which moderate boundary thresholds are more analytically productive than extreme cutoffs. The categorization scheme is presented in Table 3.

Table 3.

Student Performance Categorization

Category	Score Range
High ability	$x \geq \mu + 0.5\sigma$
Medium ability	$\mu - 0.5\sigma \leq x < \mu + 0.5\sigma$
Low ability	$x < \mu - 0.5\sigma$

These categories were used analytically to identify contrasting patterns of engagement with numerical constraints. Rather than representing fixed ability levels, the categories functioned as analytical lenses for examining differences in students' numerical reasoning and epistemic strategies during cryptarithmic problem solving.

Qualitative Analysis: Interpreting Numerical Reasoning and Justification

Qualitative data from student and teacher interviews were analyzed using Thematic Analysis following the six-phase framework proposed by Braun and Clarke (2006): familiarization, initial coding, theme generation, theme review, theme definition, and reporting. This method was chosen for its flexibility in capturing patterns of meaning related to reasoning, justification, and epistemic regulation in complex problem-solving contexts (Nowell et al., 2017). In the familiarization phase, interview transcripts were read repeatedly to develop a deep understanding of the data as a whole before any coding began. Initial codes were then

generated inductively by annotating segments of data that described how students and teachers engaged with, responded to, or interpreted cryptarithmic tasks — including observable strategies such as guessing, elimination, and pattern recognition. These initial codes were not imposed from theory but allowed to emerge from the data, consistent with an inductive, grounded approach to qualitative inquiry. In the theme generation phase, codes were grouped into candidate themes by identifying patterns of shared meaning across participants and data sources. Themes were subsequently reviewed against the full dataset to ensure they accurately represented the range of perspectives captured in the interviews, refined through iterative discussion among the research team, and formally defined to capture their epistemic significance before reporting.

To manage researcher subjectivity throughout this process, several strategies were employed. First, an audit trail was maintained in NVivo 14, documenting all coding decisions and revisions to ensure transparency and analytical consistency. Second, peer debriefing was conducted among the research team, in which coding decisions were discussed and challenged to reduce the influence of individual interpretive biases (Nowell et al., 2017). Third, member checking was carried out with selected participants, who were invited to review emerging interpretations of their responses to confirm that the themes accurately reflected their intended meanings (Creswell & Clark, 2017). Together, these procedures ensured that the thematic analysis was conducted with sufficient rigor and reflexivity to support credible interpretations of students' and teachers' epistemic engagement with cryptarithmic tasks. The validity and reliability of this study were addressed separately for its quantitative and qualitative components, in accordance with the methodological standards appropriate to each paradigm (Creswell & Clark, 2017). For the quantitative component, content validity of the cryptarithmic task set was established through expert review conducted by two mathematics education specialists, yielding a Content Validity Index (CVI) of 0.89, which exceeds the minimum threshold of 0.80 recommended for research instruments (Cohen et al., 2018). Internal consistency reliability was assessed using Cronbach's alpha, which yielded a value of 0.78, indicating adequate reliability for an exploratory instrument designed to elicit variation in reasoning rather than measure a single latent construct (Belay et al., 2024; Field, 2018). Construct validity was further supported by the theoretical alignment between the task design and the epistemic framework: each task was deliberately structured to operationalize constraint-based reasoning demands, ensuring that performance variation would reflect differences in epistemic engagement rather than incidental features of task presentation.

For the qualitative component, trustworthiness was ensured through four criteria adapted from Lincoln and Guba (1985): credibility, transferability, dependability, and confirmability. Credibility was established through prolonged engagement with the data, member checking with selected participants, and peer debriefing among the research team to challenge interpretive assumptions (Nowell et al., 2017). Transferability was supported by providing thick descriptions of the research context, participant characteristics, and task

conditions, enabling readers to assess the applicability of findings to other settings (Patton, 2015). Dependability was addressed through the maintenance of an audit trail in NVivo 14, documenting all coding decisions and analytical revisions to ensure the consistency and transparency of the analytical process. Confirmability was established by grounding all interpretive claims in direct evidence from interview transcripts and task performance data, ensuring that findings reflect participants' perspectives rather than researcher assumptions. Together, these procedures ensured that both components of the study met appropriate standards of rigor for mixed-method research.

RESULTS

Performance Patterns as Epistemic Signals

This subsection reports students' performance patterns in cryptarithmic tasks as epistemic signals of how numerical reasoning unfolds under constraint-based problem-solving conditions. Rather than interpreting scores as indicators of arithmetic ability, performance variation is examined to reveal differences in students' engagement with numerical constraints embedded in cryptarithmic tasks.

Table 4.

Descriptive Statistics of Students' Performance in Cryptarithmic Tasks

Statistic	Value
Mean	54.4 (on a 0-100 scale)
Standard Deviation (SD)	32.8
Minimum Value	15
Maximum Value	95
Median	52.5
Mode	45

Descriptive analysis of the cryptarithmic test results (Table 4) shows substantial dispersion in student performance. Scores ranged from 15 to 95 on a 0–100 scale, with a mean of 54.4, a median of 52.5, and a standard deviation of 32.8. The high standard deviation and variation coefficient (60.3%) indicate pronounced heterogeneity in how students navigated the constraints of the tasks. This wide spread suggests that students did not merely differ in accuracy, but in the ways, they coordinated digit assignments, monitored consistency, and regulated certainty while solving cryptarithmic problems.

Importantly, the coexistence of very low and near-perfect scores within the same instructional context implies that cryptarithmic tasks functioned as sensitive epistemic probes, differentiating students' numerical reasoning beyond what routine arithmetic problems typically reveal. Students with higher scores appeared to sustain coherent reasoning across multiple constraints, whereas students with lower scores frequently encountered breakdowns when numerical conflicts emerged. Thus, the observed performance dispersion reflects

variation in epistemic engagement with constraint-based reasoning rather than uniform difficulty of the tasks themselves.

Variation in Numerical Reasoning Under Constraint Conditions

To further examine how students' reason numerically when confronted with constraint-based problems, students' performance in cryptarithmic tasks was categorized using a normal distribution criterion ($\mu \pm 0.5\sigma$). Importantly, this categorization was not intended to rank students by mathematical ability, but to serve as an analytic device for tracing variation in numerical reasoning orientations under conditions where solutions cannot be obtained through routine computation. The resulting distribution of performance profiles is presented in Table 5.

Table 5.

Distribution of Students' Performance Profiles in Cryptarithmic Tasks

Performance Profile	Score Range	Number of Students	Percentage
High performance	≥ 70.8	7	25.9%
Medium performance	54.4–70.7	8	29.6%
Low performance	< 54.4	12	44.5%

Note. Performance profiles are treated as epistemic indicators reflecting dominant ways of coordinating numerical constraints in cryptarithmic tasks, rather than as fixed measures of mathematical ability.

The distribution in Table 5 reveals a pronounced asymmetry in students' engagement with cryptarithmic constraints. Nearly half of the students (44.5%) fell within the low-performance profile, indicating that for a substantial proportion of participants, maintaining consistency across multiple numerical constraints posed persistent epistemic challenges. These students frequently encountered conflicts between digit assignments and operational rules, suggesting difficulty in anticipating the global consequences of local numerical decisions. Students in the low-performance profile frequently encountered conflicts between digit assignments and operational rules, suggesting difficulty in anticipating the global consequences of local numerical decisions. This pattern is consistent with findings on ontogenic obstacles in mathematics, wherein students' cognitive unreadiness prevents systematic coordination of operational rules — a difficulty observed across varied mathematical domains (Nopriana et al., 2024). In contrast, the smaller group of students classified in the high-performance profile (25.9%) demonstrated sustained control over constraint coordination throughout the tasks. Their performance reflects an ability to reason beyond isolated operations by monitoring relational structures among digits, place values, and operational outcomes. Crucially, this difference does not merely signal higher accuracy, but points to qualitatively different modes of numerical reasoning, in which constraints are treated as governing structures rather than obstacles to be overcome through trial.

Students in the medium-performance profile (29.6%) occupied an intermediate epistemic position. While they were generally able to apply individual constraints correctly, their reasoning often remained fragmented, leading to partial solutions that broke down when

multiple constraints interacted. This pattern suggests a transitional form of numerical reasoning, in which students recognize the importance of constraints but have not yet developed strategies for coordinating them holistically.

Taken together, the uneven distribution across performance profiles indicates that cryptarithmic tasks expose substantial qualitative variation in how elementary students reason with numbers under constraint-based conditions. Rather than reflecting differences in arithmetic proficiency alone, these profiles capture differences in epistemic orientation, specifically, how students interpret constraints, anticipate numerical consequences, and regulate the validity of their solutions. This finding provides a critical foundation for identifying distinct epistemic levels of numerical reasoning, which are examined in the following subsection.

Teachers' Epistemic Interpretations and Mediation of Cryptarithmic Tasks

This subsection examines teachers' perspectives not merely as perceptions of an instructional innovation, but as epistemic interpretations that shape how cryptarithmic functions as a task for revealing students' numerical reasoning. Teachers' familiarity, perceived benefits, implementation challenges, and recommendations are analyzed as indicators of how teachers understand, mediate, and sometimes constrain the epistemic potential of cryptarithmic in classroom contexts.

Familiarity with Cryptarithmic as an Epistemic Concept

Findings from the focus group discussion indicate limited explicit familiarity with the term *cryptarithmic* among participating teachers. Only one teacher (25%) reported prior exposure to the concept, while the majority (75%) encountered the term for the first time during the study. However, this limited terminological familiarity contrasts with teachers reported instructional practices. Two teachers (50%) acknowledged having previously used similar non-routine numerical puzzles without identifying them as cryptarithmic.

From an epistemic perspective, this discrepancy points to a gap between epistemic practice and epistemic articulation. Teachers were able to enact tasks that implicitly require constraint-based reasoning, yet lacked a conceptual framework to recognize and intentionally leverage these tasks as epistemic tools. This finding suggests that the epistemic potential of cryptarithmic may remain underutilized when teachers engage with such problems intuitively rather than through an explicit understanding of their reasoning demands.

Teachers' Perceived Epistemic Functions of Cryptarithmic

Despite limited prior familiarity with the concept, all participating teachers (100%) articulated positive interpretations of cryptarithmic's value in mathematics learning. Importantly, teachers did not frame its benefits solely in terms of engagement or enjoyment, but emphasized its capacity to expose students' ways of thinking. Teachers described cryptarithmic as requiring students to connect symbols, operations, and numerical relationships, thereby preventing reliance on routine calculation. Teachers also identified cryptarithmic as a diagnostic task, noting that students' solution processes made visible differences in logical

reasoning, persistence, and error patterns. Teachers also identified cryptarithmic as a diagnostic task, noting that students' solution processes made visible differences in logical reasoning, persistence, and error patterns (Pule & Bhagwonparsadh, 2025). From an epistemic standpoint, these interpretations position cryptarithmic as a task that reveals how students justify numerical decisions and evaluate correctness, rather than merely whether answers are correct. Teachers' recognition of its role in developing higher-order thinking further reflects an implicit understanding that cryptarithmic demands epistemic engagement beyond procedural fluency.

Epistemic Challenges and Breakdown in Classroom Implementation

Teachers' accounts of implementation challenges provide insight into moments where students' epistemic engagement breaks down. Teachers observed that while some students responded enthusiastically to cryptarithmic tasks, others experienced confusion and frustration. Rather than reflecting affective resistance alone, these responses were frequently associated with difficulties in coordinating multiple numerical constraints simultaneously.

Teachers noted that students often struggled when local digit assignments conflicted with global problem consistency, leading to repeated revisions or abandonment of solution attempts. Such observations suggest that implementation challenges are rooted in epistemic regulation difficulties, where students are unable to anticipate the implications of numerical decisions across the entire problem structure. These findings reinforce the interpretation of cryptarithmic as a task that surfaces epistemic limits in students' numerical reasoning, rather than exposing simple computational weaknesses.

At the instructional level, teachers identified curriculum simplification, limited instructional time, and lack of supporting materials as factors that constrained their ability to sustain students' epistemic engagement. These constraints restricted opportunities for extended reasoning, reflection, and justification, thereby shaping how cryptarithmic was enacted in practice.

Teachers' Recommendations as Epistemic Scaffolding Strategies

Teachers' recommendations for implementing cryptarithmic reflect an emerging awareness of the need for epistemic scaffolding. Suggestions such as professional development, gradual sequencing of task difficulty, and integration with place value and whole-number topics indicate recognition that students' engagement with cryptarithmic requires intentional support. Rather than proposing cryptarithmic as an isolated enrichment activity, teachers emphasized alignment with core numerical concepts to strengthen students' reasoning foundations. From an epistemic perspective, these recommendations highlight teachers' roles as mediators who can either amplify or limit the epistemic affordances of cryptarithmic tasks. Teachers' calls for training and instructional resources suggest that effective epistemic mediation depends on teachers' ability to recognize cryptarithmic not merely as a puzzle, but as a structured environment for reasoning, justification, and constraint coordination.

Students' Epistemic Engagement, Breakdown, and Regulation in Cryptarithmic Tasks

This subsection examines students' experiences with cryptarithmic as manifestations of epistemic engagement, focusing on how students interpret the task, respond to its epistemic demands, and regulate their reasoning under constraint-based problem-solving conditions. Rather than treating students' responses as affective reactions alone, the analysis interprets familiarity, emotions, perceived benefits, and difficulties as indicators of how students engage with, struggle with, and adapt to the epistemic structure of cryptarithmic tasks.

Initial Epistemic Framing of Cryptarithmic

Interviews with six students revealed that none of the participants (100%) had prior familiarity with the term *cryptarithmic*. However, students' informal descriptions, such as referring to the task as a "letter but number" problem, indicate an emerging epistemic framing of the task as one involving symbolic substitution rather than routine calculation. Although only two students (33.3%) reported having encountered similar problems previously, all students recognized cryptarithmic as qualitatively different from standard arithmetic tasks, suggesting that novelty arose not merely from unfamiliar terminology but from the task's epistemic demands. This initial framing is significant from an epistemic perspective, as it reflects how students first conceptualize what kind of mathematical activity is required. Students' spontaneous descriptions suggest an early awareness that solving cryptarithmic requires reasoning about relationships between symbols and numbers, rather than direct computation.

Emotional Responses as Indicators of Epistemic Engagement and Breakdown

Students' emotional responses to cryptarithmic varied considerably, ranging from enjoyment to frustration. Two students (33.3%) described the task as enjoyable and engaging, while two others (33.3%) expressed mixed feelings, noting enjoyment only when partial progress was achieved. The remaining two students (33.3%) reported negative experiences, describing feelings of confusion and frustration.

From an epistemic standpoint, these emotional responses signal different levels of engagement with the task's reasoning demands. Positive emotions were often associated with moments of epistemic coherence, when students perceived consistency among digit assignments and constraints. In contrast, frustration and confusion coincided with epistemic breakdowns, where students were unable to reconcile conflicting constraints or anticipate the consequences of numerical decisions. Thus, emotions functioned as indicators of students' success or difficulty in regulating their epistemic activity, rather than as isolated affective reactions.

Students' Perceived Epistemic Value of Cryptarithmic

Despite experiencing difficulty, students articulated several perceived benefits of engaging with cryptarithmic. They reported that the task encouraged critical thinking, logical reasoning, concentration, and sustained attention. Students also described cryptarithmic as sharpening problem-solving skills and providing a meaningful challenge that extended beyond routine classroom activities. These perceptions suggest that students recognized cryptarithmic as an

epistemically valuable task, one that requires deliberate reasoning and reflection. Importantly, students' appreciation of these benefits emerged even in the presence of difficulty, indicating that epistemic value was not contingent on immediate success but on engagement with the reasoning process itself.

Epistemic Difficulties and Regulation Strategies

Students identified several difficulties when working on cryptarithmic tasks, particularly when repeated attempts failed to produce consistent solutions. Feelings of disappointment and self-doubt were commonly reported during these moments. Students also emphasized the importance of gradual learning and expressed a preference for guided support before working independently. Interpreted epistemically, these difficulties reflect challenges in epistemic regulation, specifically in monitoring consistency, revising assumptions, and managing uncertainty under constraint-based conditions. Students' calls for step-by-step guidance indicate an emerging awareness of the need for external scaffolding to support epistemic transitions, from trial-based reasoning toward more structured coordination of constraints. These findings highlight that effective engagement with cryptarithmic requires not only cognitive effort but also support mechanisms that help students regulate their reasoning processes productively.

Structural Constraint Interaction and Limits of Numerical Reasoning in Cryptarithmic

This subsection examines how the **structural interaction of numerical constraints** within cryptarithmic tasks shapes students' numerical reasoning and exposes its epistemic limits. Rather than attributing performance differences to arithmetic operation types or general ability, the analysis foregrounds how students respond when multiple constraints interact in ways that cannot be resolved through local or sequential reasoning.

Differential Effects of Constraint Interaction Across Task Structures

Table 6 presents students' success rates across cryptarithmic tasks with different arithmetic structures. To examine whether the observed differences in success rates across task types were statistically meaningful rather than attributable to sampling variation, a Cochran's Q test was conducted, as this non-parametric test is appropriate for comparing proportions across three or more related conditions within the same sample (Field, 2018). The result was statistically significant ($Q = 18.47$, $df = 3$, $p < .001$), confirming that success rates differed systematically across task types beyond what would be expected by chance. Post-hoc pairwise comparisons using McNemar's test with Bonferroni correction revealed that the difference between 3-digit addition (29.6%) and 2-digit multiplication (70.4%) was the most pronounced and statistically reliable ($p < .01$), while the difference between 2-digit addition (88.9%) and 3-digit addition (29.6%) was also significant ($p < .001$). These results confirm that the performance paradox observed in the data reflects a genuine structural effect of constraint interaction rather than a product of the small sample size.

A critical phenomenon emerges from these results: students' performance does not align with conventional hierarchies of arithmetic difficulty. Although addition is typically considered

less complex than multiplication, students experienced the most severe breakdowns in 3-digit addition. This finding indicates that difficulty in cryptarithmic is not determined by operation type, but by the degree to which numerical constraints interact across positional structures.

Table 6.

Student Success Rates by Operation Type

Operation Type	Correct	Wrong	Correct (%)
2-digit addition	24	3	88.9%
3-digit addition	8	19	29.6%
2-digit subtraction	15	12	55.6%
2-digit multiplication	19	8	70.4%

In 3-digit addition, constraints related to digit uniqueness, place value, and carrying are tightly coupled. A decision made in one column immediately propagates consequences across other columns, requiring students to anticipate interactions between local assignments and global consistency. For many students, this interaction exceeded the limits of their numerical reasoning, leading to frequent contradictions and abandoned solution paths. In contrast, 2-digit multiplication, despite involving more complex computation in routine arithmetic, presented a more stable constraint structure in cryptarithmic form, allowing students to reason about digit relations without extensive cross-column propagation. This reversal, where addition becomes more epistemically demanding than multiplication, constitutes a structural constraint interaction effect, revealing how cryptarithmic tasks amplify the limits of students' ability to coordinate numerical relationships under interdependent conditions.

Epistemic Limits Revealed by Performance Profiles

Analysis across performance profiles further clarifies how students differed in their capacity to manage interacting constraints. Students in the high-performance profile demonstrated the ability to maintain global consistency across multiple constraint interactions. Their reasoning extended beyond sequential elimination, incorporating anticipatory judgments about how digit assignments would affect other components of the problem structure.

Students in the medium-performance profile exhibited partial control over constraint interaction. While they could manage isolated constraints, their reasoning frequently collapsed when multiple constraints interacted simultaneously, particularly in tasks involving carrying. This pattern suggests an epistemic threshold where students' reasoning remains locally coherent but globally unstable. Students in the low-performance profile consistently encountered epistemic limits when constraint interaction intensified. Their strategies relied heavily on empirical trial-and-error, and they rarely anticipated the cascading effects of numerical decisions. Breakdown occurred not because students failed to understand individual rules, but because they lacked mechanisms for coordinating constraint interaction as a system. Across profiles, performance differences thus reflect qualitative limits in numerical reasoning under interacting constraints, rather than gradations of arithmetic skill.

Cryptarithmic as a Revealer of Epistemic Thresholds

Taken together, these findings demonstrate that cryptarithmic tasks function as revealing instruments of epistemic thresholds in elementary numerical reasoning. When constraints operate independently, many students are able to sustain coherent reasoning. However, as constraint interaction intensifies, particularly through place-value coupling and carry propagation, students' reasoning reaches identifiable limits. These results show that cryptarithmic does not merely differentiate students by correctness, but exposes where and how numerical reasoning breaks down under structurally complex conditions. Such breakdowns make visible the epistemic boundaries of students' reasoning, providing empirical grounding for interpreting cryptarithmic as an epistemic task rather than an assessment of procedural competence.

To further substantiate the qualitative findings, the prevalence and diversity of themes identified through thematic analysis are summarized here. Across all six student interviews, three themes emerged with high prevalence: epistemic breakdown under constraint interaction (present in all six participants, 100%), reliance on trial-and-error as a primary reasoning strategy (five of six participants, 83.3%), and awareness of task novelty as epistemically distinct from routine arithmetic (all six participants, 100%). Two additional themes emerged with moderate prevalence: emotional regulation as an indicator of epistemic coherence or collapse (four of six participants, 66.7%), and recognition of the need for external scaffolding to sustain reasoning (four of six participants, 66.7%). Across teacher focus group data, three themes were consistently represented: cryptarithmic as a diagnostic tool for revealing reasoning quality (all four teachers, 100%), implementation challenges rooted in curriculum and time constraints (three of four teachers, 75%), and perceived need for professional development and epistemic scaffolding strategies (all four teachers, 100%). The diversity of these themes — spanning cognitive, affective, and pedagogical dimensions — confirms that the thematic analysis captured a substantively broad range of epistemic phenomena rather than a narrow or redundant set of observations.

Epistemic Integration Across Multiple Perspectives

This subsection integrates findings from quantitative performance data, student interviews, and teacher focus group discussions to construct a coherent epistemic account of how cryptarithmic functions as a task for revealing elementary students' numerical reasoning under constraint-based problem-solving conditions. Rather than using triangulation solely to confirm consistency, this integration highlights how convergence and divergence across perspectives jointly illuminate the epistemic structure, demands, and limits of cryptarithmic tasks.

Epistemic Convergence Across Data Sources

A strong epistemic convergence was observed across teachers' interpretations, students' experiences, and quantitative performance indicators. All data sources independently revealed substantial variation in students' engagement with cryptarithmic tasks, particularly when

numerical constraints interacted in complex ways. Teachers reported noticeable differences in students' reasoning quality, students described tasks as either manageable or highly confusing, and quantitative results showed clear performance differentiation across tasks with varying constraint density. This convergence is summarized in Table 7.

Table 7.

Epistemic Convergence Across Data Sources in Cryptarithmic Tasks

Epistemic Aspect	Teacher Interpretation	Student Experience	Quantitative Indicator
Variation in numerical reasoning	Students exhibit diverse reasoning quality	Tasks perceived as easy or very difficult	Distinct performance profiles (high–medium–low)
Constraint interaction difficulty	Complex tasks cause widespread difficulty	Confusion when constraints conflict	Sharp decline in success rates with increased constraint density
Sustained epistemic engagement	Approximately 20–30% of students persist productively	Only some students report enjoyment	High-performance profile: 25.9%

Note. Convergence across data sources indicates that performance variation reflects differences in epistemic engagement with numerical constraints rather than arithmetic ability alone.

As shown in Table 7, convergence across perspectives confirms that cryptarithmic tasks consistently expose variation in how students coordinate, justify, and regulate numerical reasoning under constraint-based conditions. Importantly, teachers' estimations regarding the proportion of students who engage productively with cryptarithmic closely align with the proportion of students in the high-performance profile, reinforcing the interpretation of cryptarithmic as a sensitive epistemic probe rather than a conventional achievement measure.

Epistemic Divergence in Interpreting Cryptarithmic Tasks

Alongside convergence, epistemic divergence emerged in how teachers and students interpreted the function and meaning of cryptarithmic tasks. While teachers predominantly framed cryptarithmic as a diagnostic tool for revealing students' reasoning quality and strategy use, students interpreted the same tasks primarily through experiential and affective lenses, viewing them as challenges or game-like activities that elicited enjoyment or frustration. This divergence does not imply misalignment regarding the task itself, but rather highlights how epistemic meaning is shaped by participants' roles. Teachers evaluate cryptarithmic through the lens of instructional insight, whereas students experience it as a demanding epistemic activity whose success or failure is felt affectively. These differing epistemic positions influence

engagement and persistence without altering the underlying constraint-based structure of the task. This divergence is presented in Table 8.

Table 8.

Epistemic Divergence in Interpreting Cryptarithmic Tasks

Epistemic Dimension	Teacher Perspective	Student Perspective
Epistemic function of the task	Diagnostic tool to reveal reasoning quality	Challenge or game-like activity
Criterion of correctness	Coherence and consistency of reasoning	Emotional resolution (feels solvable or frustrating)
Focus of engagement	Evaluation of reasoning processes	Immediate experience of difficulty or enjoyment
Perceived value	Insight into students' numerical reasoning	Enjoyment, challenge, or avoidance

Note. Divergence reflects differences in epistemic positioning rather than misunderstanding of task goals.

Integrated Epistemic Phenomena Revealed Through Triangulation

Integrating findings across perspectives reveals epistemic phenomena that are not fully visible within any single data source. The paradox in which students demonstrated higher success rates in two-digit multiplication than in three-digit addition illustrates this integrative value. Quantitative data identify the performance paradox, student interviews reveal moments of confusion and breakdown during carrying operations, and teacher observations contextualize these difficulties as failures of constraint coordination rather than procedural knowledge. Similarly, the identification of multiple problem-solving strategies, ranging from trial-and-error to elimination and pattern recognition, gains epistemic significance when linked across datasets. Trial-based strategies were frequently associated with emotional breakdown and disengagement, whereas pattern recognition aligned with sustained engagement and coherent reasoning. These patterns demonstrate that strategy use, emotional regulation, and numerical reasoning are epistemically intertwined, forming a unified system of engagement with constraint-based tasks.

Cryptarithmic as an Epistemic Revealer of Numerical Reasoning

Taken together, the integrated findings demonstrate that cryptarithmic functions as a revealing epistemic task that makes elementary students' numerical reasoning observable across performance patterns, experiential responses, and interpretive judgments. Convergence across data sources confirms the robustness of this interpretation, while divergence clarifies how epistemic meaning is negotiated differently by teachers and students. By integrating multiple perspectives, this study shows that cryptarithmic creates an epistemic space in which numerical reasoning becomes visible through moments of coherence, breakdown, and regulation under interacting constraints. These results provide a coherent empirical foundation

for conceptualizing cryptarithmic not merely as a puzzle or assessment tool, but as an epistemic task that systematically reveals how students reason with numbers when routine procedures are insufficient.

DISCUSSION

This study advances mathematics education research by repositioning cryptarithmic from a peripheral non-routine exercise to a central epistemic task for examining elementary students' numerical reasoning under constraint-based problem-solving conditions. Rather than contributing incremental evidence about problem-solving performance or higher-order thinking skills, the findings offer a conceptual shift: numerical reasoning becomes visible not through correctness, but through how students coordinate constraints, manage uncertainty, and regulate breakdowns when routine procedures no longer suffice.

The integration of quantitative performance patterns, student experiences, and teacher interpretations demonstrates that variation in outcomes reflects differences in epistemic engagement, not merely differences in arithmetic proficiency. Performance dispersion, emotional responses, and strategic diversity collectively indicate that cryptarithmic tasks operate as epistemic stress tests, exposing the limits and organization of students' numerical reasoning. In this sense, triangulation in the present study functions as a theory-building strategy, revealing reasoning structures that remain hidden in conventional assessments focused on procedural accuracy. This supports broader claims in educational research that complex cognitive phenomena require multi-perspective approaches to be adequately theorized (Flick, 2018; Holzer et al., 2023).

A key theoretical insight emerging from this study concerns the role of constraint interaction in shaping numerical reasoning. The finding that three-digit addition posed greater epistemic difficulty than two-digit multiplication challenges entrenched assumptions about arithmetic complexity hierarchies. Rather than operation type, it is the density and interdependence of constraints, place value coordination, digit uniqueness, and carrying operations, that govern epistemic load. This extends cognitive load theory by emphasizing that epistemic difficulty arises not only from information quantity, but from the necessity to anticipate and coordinate interacting constraints across representational levels (Foster, 2023; Torres-Peña et al., 2024). Cryptarithmic thus provides a powerful context for identifying epistemic thresholds, where reasoning coherence collapses under constraint pressure.

The pronounced heterogeneity in students' performance and emotional responses further underscores that numerical reasoning is inseparable from epistemic regulation. Frustration, enjoyment, and disengagement should not be interpreted as affective by-products, but as epistemic signals indicating moments of breakdown or coherence. Students who relied heavily on trial-and-error strategies exhibited rapid epistemic collapse, whereas those employing elimination or pattern recognition demonstrated greater stability under constraint interaction. These patterns align with research emphasizing the centrality of metacognitive

monitoring and regulation in sustaining mathematical reasoning (Mulcahy et al., 2024; Tay et al., 2024). Importantly, the observed gender-related differences in persistence and frustration are best understood as variations in epistemic regulation strategies rather than inherent cognitive differences, reinforcing the need to situate affect within epistemic activity (Van Mier et al., 2019; Wang & Abdullah, 2024).

The divergence between teacher and student perspectives further illuminates how epistemic meaning is negotiated in classroom contexts. Teachers tended to frame cryptarithmic as a diagnostic tool for revealing reasoning quality, while students experienced it as an epistemically demanding challenge accompanied by uncertainty and affective tension. This divergence reflects differing epistemic criteria: teachers privilege coherence and explanatory value, whereas students prioritize experiential resolution of uncertainty. Such findings echo research on multiple perspective interviews, which emphasizes that shared activities can carry distinct epistemic meanings depending on participants' roles (Holzer et al., 2023). Recognizing this divergence is crucial, as it shapes engagement and persistence without altering the underlying epistemic structure of the task.

From a design perspective, the findings position cryptarithmic as a form of constraint-based epistemic design that is theoretically distinct from, though complementary to, established problem-based and higher-order thinking pedagogies. Unlike instructional approaches that rely on contextual authenticity or gamification to drive engagement (Widodo et al., 2019), cryptarithmic derives its epistemic power from making reasoning processes unavoidable and visible under constraint pressure. Its effectiveness, therefore, depends less on motivational framing and more on instructional conditions that support epistemic mediation, such as explicit attention to constraint coordination and opportunities for reflective regulation. Overall, this study contributes a theoretically grounded account of how cryptarithmic functions as an epistemic task that reveals elementary students' numerical reasoning through patterns of coordination, breakdown, and regulation. By shifting attention from outcomes to epistemic processes, the findings offer a framework for designing mathematical tasks that make thinking visible in elementary classrooms. This reframing has implications beyond cryptarithmic, suggesting that constraint-based tasks can serve as powerful epistemic tools for studying and supporting numerical reasoning in mathematics education.

Limitations

Notwithstanding these contributions, several limitations of the present study warrant acknowledgment. First, the study was conducted within a single urban elementary school in Jakarta, involving a purposive sample of 27 fifth-grade students and 4 mathematics teachers. While this sample was appropriate for the exploratory and epistemic aims of the research, its restricted size and single-site character limit the transferability of findings to other school contexts, grade levels, or cultural settings (Patton, 2015). The performance patterns and reasoning orientations identified here should therefore be interpreted as contextually situated rather than broadly representative. Second, the use of a cross-sectional design means that the

study captures students' epistemic engagement at a single point in time, without examining how reasoning develops or changes across repeated exposure to cryptarithmic tasks. Longitudinal designs would be better positioned to trace epistemic transitions and the development of constraint coordination over time. Third, although member checking and peer debriefing were employed to manage researcher subjectivity, the qualitative interpretations remain inherently shaped by the researchers' theoretical orientation toward epistemic cognition. Alternative analytical frameworks may yield different, equally valid interpretations of the same data. Finally, the cryptarithmic tasks used in this study were limited to four problem types covering addition, subtraction, and multiplication. The findings regarding constraint interaction effects may not generalize to other task structures, such as division-based or multi-operation cryptarithmic problems, which may engage different reasoning demands. Future research should address these limitations by employing larger and more diverse samples, longitudinal designs, and a broader range of task structures to extend and test the theoretical account developed here.

CONCLUSION

This study set out to examine how cryptarithmic operates as an epistemic task for revealing elementary students' numerical reasoning under constraint-based problem-solving conditions. The findings show that cryptarithmic does not merely differentiate students by correctness or arithmetic skill, but makes visible how students coordinate numerical constraints, justify digit assignments, and regulate reasoning when routine procedures are insufficient.

Across tasks with increasing constraint density, students exhibited distinct reasoning orientations marked by patterns of coherence, breakdown, and recovery. Performance variation, emotional responses, and strategy use collectively indicate that difficulty in cryptarithmic is governed less by the type of arithmetic operation and more by the interaction of constraints, particularly those involving place value coordination and carrying processes. The observed paradox, where three-digit addition posed greater difficulty than two-digit multiplication, underscores this conclusion and highlights the role of constraint interaction in shaping epistemic demand.

The integration of quantitative task outcomes with student and teacher perspectives further clarifies how numerical reasoning is enacted and interpreted in classroom contexts. Teachers tended to view cryptarithmic as a diagnostic tool for accessing students' reasoning, while students experienced it as an epistemically demanding activity accompanied by uncertainty and affective tension. These differing perspectives do not signal misalignment, but rather reveal how epistemic meaning is negotiated through roles and experiences, influencing engagement without altering the underlying structure of the task. Taken together, the findings demonstrate that cryptarithmic functions as a revealing epistemic task that externalizes numerical reasoning through observable patterns of coordination, breakdown, and regulation. This study contributes to mathematics education research by offering an empirically grounded

account of how constraint-based tasks can be designed and used to make students' thinking visible in elementary classrooms. More broadly, it suggests that focusing on constraint interaction provides a productive lens for understanding and supporting numerical reasoning beyond procedural correctness.

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